

# Area Law from Loop Quantum Gravity

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(Dated: June 5, 2015)

We explore the constraints following from requiring the Area Law in the entanglement entropy in the context of loop quantum gravity. We find a unique solution to the single link wave-function in the large  $j$  limit, believed to be appropriate in the semi-classical limit. We then generalize our considerations to multi-link coherent states, and find that the area law is preserved very generically using our single link wave-function as a building block. Finally, we develop the framework that generates families of multi-link states that preserve the area law while avoiding macroscopic entanglement, the space-time analogue of “Schrödinger cat”. We note that these states, defined on a given set of graphs, are the ground states of some local Hamiltonian that can be constructed explicitly. This can potentially shed light on the construction of the appropriate Hamiltonian constraints in the LQG framework.

PACS numbers: 04.60.Pp

In recent years, the study of entanglement has shed new lights in the fields of quantum field theory, quantum statistical mechanics and condensed matter theory. Whenever quantum fluctuations are important, entanglement plays an important role [1]. In the context of condensed matter physics, it has played a key role in classifying phases of matter, notably those with mass gaps, leading to the notion of topological orders [2–5] characterized by long-range entanglement [6–9]. A crucial quantifier of quantum entanglement is the entanglement entropy. For a wave-function that is the ground state of a local Hamiltonian, it is believed that the entanglement entropy should obey the so called area law [10]: in the limit of a large subregion  $A$ , its entanglement entropy is given by  $S_A = -\text{Tr} \rho_A \ln \rho_A$ , where  $\rho_A$  is the reduced density matrix of subsystem  $A$ , and  $S_A$  scales to leading order as the area of the boundary of  $A$ .

From lessons that have been drawn from black hole entropy, which is proposed to be understood as an entanglement entropy between degrees of freedom within and without the horizon [11–13], and more recently evidence from the computation of entanglement entropy via the Ryu-Takayanagi formula [14] in the AdS/CFT correspondence, it appears that one very important feature of quantum gravity at least in the semi-classical limit, in which some classical background geometry can be defined, is that the leading contribution to the entanglement entropy associated to a sub-region  $A$  should also satisfy the area law [15].

Loop Quantum Gravity (LQG) [16–18] provides a perfect arena to test the requirement of area law, in which explicit calculations can be done. A long standing and central question in the LQG is to understand the semi-classical limit. Thus far, semi-classical states have been proposed [19–22] in LQG, and studied in many contexts [23–40]. However, it is still by far an open question whether a semi-classical geometry has been undis-

putedly recovered. The current paper is partly inspired by recent work in LQG that recovers the Bekenstein’s black hole entropy [41–50] and several attempts in computing entanglement entropy in terms of spin network states [51–55], which is intimately related to the area law of the entanglement entropy [15, 55].

In this letter, we inspect the semi-classical limit through the lens of entanglement entropy of a bounded sub-region in LQG. We find that imposing the area law across a single triangular surface, the simplest possible scenario, already leads to a very tight constraint on the possible wave-function, which admits an almost unique solution, one that acquires an interesting correction from what is envisaged in [15].

Ultimately the main goal of this paper is to show that entanglement imposes very stringent requirement to a theory of quantum gravity. We show how to use this guide in a constructive way. In particular, we impose that, together with the area-law, entanglement has to obey (i) SU(2) gauge invariance, and (ii) it must be *microscopic*, implying that fluctuations in quantum spacetimes should be confined to microscopic scales. These are very natural requirements if Entanglement is physically measurable. Gauge invariance means that entanglement does result in stronger than classical correlations in actual physical observables [56], and microscopic entanglement means that we do not allow arbitrarily large Schrödinger cat-spacetimes [57]. These considerations, together with the single-link result, allows us to construct the many-body or multi-facet state of a general quantum geometry. As a bonus, we obtain a constructive method to build local Hamiltonians for given set of graphs, potentially important towards understanding the Hamiltonian constraints of LQG.

Let us begin by setting the stage and notations. Our starting point is the SU(2) spin-network states in the graph-fixed kinematical Hilbert space  $\mathcal{H}_\Gamma$  of LQG. An

SU(2) spin-network state is a triplet  $|\mathcal{S}\rangle = |\Gamma, j_l, i_n\rangle$ .  $\Gamma$  is the given *proper* graph with  $L$  oriented links and  $N$  nodes.  $j_l \in \mathbb{Z}^+/2$  is an assignment of an SU(2) unitary irreducible representation to each link  $l$ , and  $i_n$  is an assignment of an SU(2) intertwiner to each node  $n$ . The spin- $j_l$  representation is isomorphic to a  $\text{SL}(2, \mathbb{C})$  representation  $(\gamma j_l, j_l)$  selected by the linear version of the simplicity constraints [58], namely  $\vec{K} = \gamma \vec{L}$ , in which  $\vec{K}$  and  $\vec{L}$  are boost and rotation generators, respectively. The simplicity constraints are used in the spin foam formalism [59–62] for recovering gravity amplitudes from BF(topological) amplitudes [63, 64] and for regaining the real connection [62, 65]. Spin-network states correspond to the so called *twisted geometries* [66, 67], which describe the geometries of three dimensional *fuzzy* discrete manifolds. Each  $N$ -valent node corresponds to a  $N$ -facet polyhedron, while each link is dual to the face of the polyhedron [68]. The areas of the faces are realized as the expectation value of the SU(2) Casimir operator acting on the spin-network states, *i.e.* the area of the face dual to link  $l$  is

$$\langle \hat{\mathcal{A}}_l \rangle = 8\pi\gamma\ell_p^2 \sqrt{j_l(j_l + 1)}, \quad (1)$$

where  $\gamma \in \mathbb{R}^+$  in the rest of the letter, and  $\ell_p$  is the Planck length.

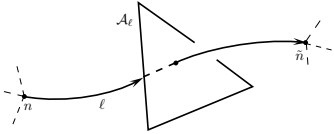


Figure 1. A face  $\mathcal{A}_l$  dual to link  $l$ .

Given a state  $|\Psi\rangle$  in  $\mathcal{H}_\Gamma$  and cut the graph  $\Gamma$  into two regions  $A$  and  $\bar{A}$ , the entanglement entropy  $S_A$  between these two regions is defined as the von Neumann entropy of the reduced density matrix  $\rho_A \equiv \text{Tr}_{\bar{A}} |\Psi\rangle \langle \Psi|$ .  $\text{Tr}_{\bar{A}}$  is the partial trace over the states (*i.e.* the intertwiners' and spins' degrees of freedom) inside  $A$ . To recover locally a Minkowskian vacuum that respects the Bisognano-Wichmann theorem, it is proposed that the reduced density matrix  $\rho_A$  should be proportional to the exponentiation of the boost operator  $K$ , leading to the following proposal for a single link state [15, 69]  $|\Psi\rangle = \sum_j \alpha_j \exp(-\gamma K)_{mn} |j, m, n\rangle$ . A careful check however suggests that it *does not* satisfy the area law [70] in the large  $j$  limit, which is another crucial ingredient of the semi-classical limit, opposing naive expectation. Indeed, one can readily check that the normalization of the reduced density matrix always cancel the area term in this limit. Surprisingly, in the large  $j$  regime, there is a natural and arguably unique solution that recovers the area law across each link. For a single link we can conveniently choose a state in the diagonalized basis with fixed

spin  $j$  such that the reduced density matrix satisfies

$$\rho_A \equiv \sum_{m=-j}^j p(m) |j, m\rangle \langle j, m|, \quad p(m) \equiv \frac{f(m)}{\sum_{m=-j}^j f(m)}.$$

The function  $f(m)$  is assumed not to carry explicit  $j$  dependence. We will also replace the sum by an integral  $\sum_{m=-j}^j \rightarrow \int_{-j}^j dj$ , assuming that the distribution  $p(m)$  is negligible at small values of  $|m|$  in the large spin limit, an expectation that is naturally born out in the original proposal of Ref. [69]. Requiring that the entropy admits an area law and taking into account (1), we impose  $S_A = cj + \dots$ , the derivative of which w.r.t.  $j$  is thus a constant  $c > 0$ . This leads to a constraint on  $f(m)$  [71]:

$$\frac{c}{p(j)} + cj - 1 + \ln p(j) = 0. \quad (2)$$

Eq. (2) cannot be solved exactly in complete generality. However, anticipating that we are considering the large spin limit, and motivated by the original proposal, we consider the limit in which  $c/p(j) \ll |cj - 1|$ . In this limit, we have  $p(j) = \exp(1 - cj)$ , which finally gives

$$f(m) = \exp \left[ -cm - \frac{\exp(1 - cm)}{c} \right]. \quad (3)$$

One can readily check that  $c/p(j) \ll |cj - 1|$  is satisfied in the large  $j$  limit if we require that  $c$  scales as

$$0 < j^{-1} \ll c \ll j^{-\frac{1}{2}} \ll 1. \quad (4)$$

In this limit the leading term in the entanglement entropy is indeed linear in  $cj$ , recovering the long sought area law. The function  $p(m)$  also naturally suppresses contributions at small values of  $m$  in this limit, justifying the approximation that replaces the sum by the integral. It is very interesting that the state that we have found is very similar to the original proposal, up to an exponentially suppressed factor, which is negligible where  $f(m)$  actually contributes, but suppresses regions which would otherwise have contributed in the original proposal. The constant  $c$  also emerges naturally and plays precisely the same role as the Barbero-Immirzi parameter  $\gamma$ , and (4) is exactly the semi-classical large  $j$  regime in the covariant formalism of LQG [31]. Recall that the boost operator  $K_z$  was related to  $L_z$  via the simplicity constraint  $\vec{K} = \gamma \vec{L}$ , which followed from an action quadratic in the tetrad. One might be tempted to interpret the extra exponential correction we find here as a non-linear correction arising from quantum effects. Combining our proposal with the well known construction of multiple link coherent spin-network state [19–22], our new state  $|\Psi\rangle$  in  $\mathcal{H}_\Gamma$  is found to be

$$|\Psi\rangle \equiv \frac{1}{\mathcal{N}} \sum_{\{j_l\}, \{i_n\}} \prod_{l,n} \Delta_{(j_l, i_n)}^{j_l} F^{j_l} \cdot \bar{v}_{i_n} |\Gamma, j_l, i_n\rangle. \quad (5)$$

In (5) we defined:  $\Delta_{(t_l, J_l)}^{j_l} \equiv d_{j_l} e^{-(j_l - J_l)^2 t_l}$ , with  $d_{j_l} = 2j_l + 1$ ;  $J_l$ , a given spin for the link  $l$ , and  $t_l$ , similar to a heat kernel time; the Wigner matrix of the group elements that encodes the correction found in (3), i.e.

$$F^{j_l} \equiv D^{j_l} \left( g_l e^{-\pi \gamma L_z + i \phi_l L_z - \frac{\exp(1-2\pi\gamma L_z)}{4\pi\gamma}} g_l^\dagger \right), \quad (6)$$

in which  $D_{mn}^j(\dots)$  is defined as  $\langle j, m | \dots | j, n \rangle$ ;  $\phi_l$  is a phase;  $g_l$  and  $\tilde{g}_l$  are SU(2) elements;  $L_z$  is the  $\mathfrak{su}(2)$  generator in the  $z$  axis;  $v_i$  is the normalized intertwiner with  $\bar{v}_i \cdot v_{i'} = \delta_{ii'}$ . Note that this is a state that is factorizable into wave-functions of individual links up to gauge constraints.

We can extend our construction to the more complicated case of one node, with many links protruding from it. For simplicity, the graph  $\Gamma$  used in the following calculation is the graph with  $L$  out-pointing links attaching to only one node.

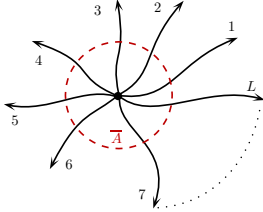


Figure 2. A graph with  $L$  links and one node.

We cut the graph into two regions as in Fig.2. Region  $\bar{A}$  contains the node. The reduced density matrix  $\rho_A$  is recovered by tracing out the link source states (or the intertwiner degrees of freedom) in  $|\Psi\rangle$ , which reads

$$|\Psi\rangle = \frac{1}{N} \sum_{\{j_l\}} \int_{SU(2)} dh \left( \prod_l d_{j_l} e^{-(j_l - J_l)^2 t_l} \right) \times \\ \times \sum_{\{m_l\}} \sum_{\{k_l\}} \prod_l F_{m_l n_l}^{j_l} D_{n_l k_l}^{j_l}(h) |j_l, m_l, k_l^\dagger\rangle. \quad (7)$$

Furthermore,  $\rho_A$  can be rewritten in the block-diagonal form  $\rho_A = \sum_{\{j_l\}} \rho_{\{j_l\}} = \sum_{\{j_l\}} P_{\{j_l\}} \hat{\rho}_{\{j_l\}}$ , where each block is characterized by the configuration of the  $\{j_l\}$  eigenvalues of the boundary links. Crucially, such a block factorization follows from the SU(2) invariance imposed at the node and the properties of the intertwiners. We can write  $P_{\{j_l\}}$ , the norm of the  $\{j_l\}$ 's block  $\rho_{\{j_l\}}$ , as

$$P_{\{j_l\}} \equiv \frac{1}{\mathcal{N}^2} \int_{SU(2)} dh \prod_l [\Delta_{(t_l, J_l)}^{j_l}]^2 \chi_{j_l}[\tilde{g}_l f(L_z) \tilde{g}_l^\dagger h]. \quad (8)$$

In (8) we use that:  $f(L_z)$  is given by (3) with  $c \equiv 2\pi\gamma$ ;  $\chi_{j_l}(M)$  is the trace over matrix  $M$ ;  $\hat{\rho}_{\{j_l\}}$  is the reduced density matrix with fixed  $\{j_l\}$

$$\hat{\rho}_{\{j_l\}} \equiv \frac{1}{Z_{\{j_l\}}} \int_{SU(2)} dh \prod_l F^{j_l} \cdot D(h) \cdot F^{j_l \dagger}, \quad (9)$$

where  $F^{j_l}$  is given by (6);  $Z_{\{j_l\}}$  is the normalization such that  $\text{Tr} \hat{\rho} = 1$ . The integration over SU(2) element  $h$  follows from the summation of intertwiner, which is proportional to an integration over Wigner matrices on SU(2)  $\sum_i \bar{v}_i^{\{p_l\}} v_i^{\{p_l'\}} \propto \int_{SU(2)} dh \prod_l D_{p_l p_l'}^{j_l}(h)$ . The normalization has been absorbed into  $\mathcal{N}$ . Then the entanglement entropy  $S_A = -\text{Tr} \rho_A \ln \rho_A$  evaluates to

$$S_A = - \sum_{\{j_l\}} P_{\{j_l\}} \text{Tr} \hat{\rho}_{\{j_l\}} \ln \hat{\rho}_{\{j_l\}} - \sum_{\{j_l\}} P_{\{j_l\}} \ln P_{\{j_l\}}.$$

Note that: i)  $P_{\{j_l\}}$  can be understood as the probability density of detecting the given boundary configuration  $\{j_l\}$ , and that  $\sum_{\{j_l\}} P_{\{j_l\}} = 1$ ; ii) it is a subtle issue to define the entanglement entropy in a gauge theory [72, 73] (moving from the proposal in [15], the resultant  $S_A$  is in fact a SU(2) gauge invariant quantity which only has dependence on the eigenvalues of the Casimir of the boundary links, a choice equivalent to the proposal in [74]); iii) the entanglement entropy with a unique boundary link-configuration is given in the first term of (10), and it will be proved later to be the area term in the large spin limit.

To calculate  $S_A$ , we first give an explicit expression for  $P_{\{j_l\}}$  and  $\text{Tr} \hat{\rho}_{\{j_l\}} \ln \hat{\rho}_{\{j_l\}}$ . As such, we introduce the coherent states on SU(2) [21], namely  $D_{mj}^j(n) |j, m\rangle = n |j, j\rangle = n |\uparrow\rangle^{\otimes 2j}$ , with  $n \in \text{SU}(2)$ . Imposing the SU(2) coherent states resolution of identity into each  $\chi_{j_l}$  in (8),  $P_{\{j_l\}}$  becomes,

$$P_{\{j_l\}} = \frac{1}{\mathcal{N}^2} \int dh \prod_l dn_l \Delta_{(t_l, J_l)}^{j_l} d_{j_l} \sum_{p_l} \frac{(-e)^{p_l}}{p_l! (2\pi\gamma)^{p_l}} e^{S_p},$$

where the “action”  $S_p$  is given by

$$S_p \equiv \sum_l 2j_l \ln \langle \uparrow | n_l^\dagger \tilde{g}_l e^{-2\pi\gamma(1+p_l)L_z} \tilde{g}_l^\dagger h | \uparrow \rangle. \quad (10)$$

In order to study the large spin behavior of  $P_{\{j_l\}}$ , we rescale  $j_l \rightarrow \lambda k_l$  and assume  $\lambda \gg 1$ . It is convenient to perform the asymptotic expansion to get the major contribution of  $P_{\{j_l\}}$ . The solutions of the equations of motion  $\delta_{n_l} S_p = \delta_h S_p = 0$  control the semi-classical behavior of  $P_{\{j_l\}}$ . For  $\delta_{n_l} S_p = 0$ , the only possible solutions [75] are  $n_l = \tilde{g}_l$ ,  $h = \mathbf{1}$ . Note that  $\delta_h S_p = 0$  is equivalent to the Gauss constraint. Then  $P_{\{j_l\}}$ , to leading order of the asymptotic expansion and in the limit of (4), becomes

$$P_{\{j_l\}} = \frac{1}{\mathcal{N}^2} \prod_l \Delta_{(t_l, J_l)}^{j_l} \frac{d_{j_l}}{2j_l} \exp \left( -1 - \frac{e^{1-2\pi\gamma j}}{2\pi\gamma} \right). \quad (11)$$

In order to estimate the term  $\text{Tr} \hat{\rho}_{\{j_l\}} \ln \hat{\rho}_{\{j_l\}}$ , it is easier to use the replica trick to compute  $S_n$ , the Rényi entropy of order  $n$  [76]. From  $\text{Tr} \hat{\rho}_{\{j_l\}}^n = \prod_l j_l^n / (n j_l) e^{(n-1)(1-2\pi\gamma j_l)}$  and taking the  $n \rightarrow 1$  limit, we get  $-\text{Tr} \hat{\rho}_{\{j_l\}} \ln \hat{\rho}_{\{j_l\}} = \sum_l 2\pi\gamma j_l - \ln \left( \prod_l j_l \right)$ . Finally, we have

$$S_A = \sum_l^L \langle 2\pi\gamma j_l \rangle - \left\langle \ln \left( \prod_l^L j_l \right) \right\rangle - \langle \ln P_{\{j_l\}} \rangle, \quad (12)$$

where we have denoted  $\langle \dots \rangle \equiv \sum_{\{j\}} (\dots) P_{\{j\}}$ . The first term is the area  $\mathcal{A}$  of region  $A$ 's boundary. The logarithmic correction is coming from both the second and third terms. When  $t_l = J_l^{-k}$ , the logarithmic correction is  $(-k+2) \ln \prod_l J_l$ . So in the semi-classical limit of LQG (4), the entanglement entropy is

$$S_A = \frac{\mathcal{A}}{4\ell_p^2} + \mu L - (k-2) \ln \left( \prod_l^L J_l \right), \quad (13)$$

where the ‘‘chemical potential’’ term now carries only  $1/j$  suppressed contributions. We would like to generalize our construction to multi-node states. Let us start with a simple state that still satisfies the area law by considering just the tensor product of single node states over all the nodes of the graph.

$$|\Psi_N\rangle = \sum_{\{j_l\}, \{i_n\}} c_{\{j_l\}\{i_n\}} \prod_n^N |\Psi_n\rangle, \quad (14)$$

$$|\Psi_n\rangle \equiv |\{j_l \in n\}, i_n\rangle, \quad c_{\{j_l\}\{i_n\}} \equiv \frac{1}{N} \prod_{l,n} \Delta_{(t_l, J_l)}^{j_l} F^{j_l} \cdot \bar{v}_{i_n}.$$

The links that are completely enclosed within the region  $\bar{A}$  would not contribute to the entanglement entropy, which will instead scale with the number of links crossing the boundary  $\partial\bar{A}$  yielding the area law,  $\mathcal{A} = \sum_{l \in \partial\bar{A}} \langle \hat{A}_l \rangle$ . From the many-body point of view, this state is trivial, as there are no quantum fluctuations other than the single-body ones. A generic superposition of such kind of states, though, will first of all not respect the area law, and will contain arbitrarily macroscopic superpositions. We need to deform the trivial state in a way that some entanglement and quantum fluctuations are produced, but without violating the constraints we set up. This can be obtained by the technique of *quasi-adiabatic continuation* introduced in [77]. This technique allows to continuously deform a quantum state that is the ground state of a local Hamiltonian in order that it is still the ground state of a local Hamiltonian. Let  $H(\lambda)$  be a smooth family of *local* Hamiltonians parametrized by  $\lambda \in [0, 1]$  such that  $|\Psi_N\rangle = |\Psi(0)\rangle$  is the ground state of  $H(\lambda=0)$ . Here, local means that  $H(\lambda)$  is the sum of local operators,  $H(\lambda) = \sum_X \hat{T}_X(\lambda)$ , where  $X$  denotes a subset of the graph  $\Gamma$  and  $\hat{T}_X(\lambda)$  means that this operator has only support on  $X$ . Also assume that  $H(\lambda)$  has a finite gap  $\Delta E$  between ground and first excited state for all  $\lambda$ 's. One way to think of  $H(\lambda)$  is as the perturbation of the initial Hamiltonian  $H(0)$ , namely,  $H(\lambda) = H(0) + \lambda \sum_X \hat{K}_X$  and  $\hat{K}_X$  are local operators with support on  $X$ . Then, following [78], one can define a unitary operator  $U(\lambda) = \mathcal{T} \exp\{-i \int_0^\lambda \tilde{H}(s) ds\}$  with  $\mathcal{T}$  the time ordering operator, and  $\tilde{H}(s) = i \int dt F(t) \exp(iH(s)t) (\partial_s H(s)) \exp(-iH(s)t)$ , with  $F(t)$  an appropriate fast decaying smooth function (see [78]

for details). The unitary operator  $U(\lambda)$  so defined interpolates among ground states of  $H(\lambda)$  and it is called *adiabatic continuation* [77]. The ground state of  $H(\lambda)$ ,  $|\Psi(\lambda)\rangle$ , can be written as the adiabatic continuation of  $|\Psi(0)\rangle$ , namely  $|\Psi(\lambda)\rangle = U(\lambda) |\Psi(0)\rangle$ . Now, the adiabatic continuation preserves the area law [10]. In this way, we can always deform  $|\Psi(0)\rangle$  in a way to obtain a new state  $|\Psi(\lambda)\rangle$  such that its parent Hamiltonian  $H(\lambda)$  is non-integrable.

We now show that by adiabatic continuation we also preserve the physicality of entanglement as being microscopic. Adiabatic continuation preserves macroscopic entanglement. So, if one continues a state that is not a Schrödingers' cat, one will not obtain a Schrödingers' cat. On the other hand, continuing a Schrödingers' cat will still yield a macroscopically entangled state. To show this, we apply a result obtained in [57]. The superselection rule is imposed by stating that states with non vanishing mutual information  $\mathcal{I}_\infty$  between two distant macroscopic regions must be ruled out. The mutual information is defined as  $\mathcal{I}(A|B) := S(A) + S(B) - S(AB)$ . Here,  $S$  is chosen to be the 2-Rényi entropy  $S_2 = -\log \text{Tr} \rho_A^2$ , namely, the logarithm of the purity instead the von Neumann entropy. The technical reason for making this choice is that one can prove that  $\mathcal{I}_\infty$  is preserved by quasi adiabatic continuation. The physical reason, is that this quantity is a physical observable (unlike the Von Neumann Entropy) [79–81], but still captures all the entanglement properties that are important in quantum many-body theory [82, 83]. As a bonus, starting with a trivial non-interacting Hamiltonian that is the sum of commuting local terms in the tensor product structure of Eq.(14), one can adiabatically continue the very Hamiltonian by means of any other Hamiltonian with a gap, which is the sum of local operators with couplings depending on some parameter  $\lambda$ . This is thus a systematic construction of local Hamiltonians whose ground states satisfy the desired properties we have set forth.

To conclude, we have shown that entanglement can provide stringent guiding principles in selecting states with sensible semiclassical limits in LQG. We obtain these results by imposing that entanglement must be physical (gauge invariant and not macroscopic) by using methods from quantum information and quantum many-body theory. In perspective, the framework here developed will allow for the study of notions like quantum order or thermalization in a closed quantum system in the context of quantum gravity.

This work was supported in part by the National Basic Research Program of China Grant 2011CBA00300, 2011CBA00301 the National Natural Science Foundation of China Grant 61033001, 61361136003.

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